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Dimensional reduction of ten-dimensional supersymmetric gauge theories in the $\mathcal{N}=1$, $D=4$ superfield formalism

Pantelis Manousselis and George Zoupanos

Department of Engineering Sciences, University of Patras
26110 Patras, Greece, and
Physics Department, National Technical University
Zografou Campus, 15780 Athens, Greece
E-mail: pman@central.ntua.gr, george.zoupanos@cern.ch

Abstract: A ten-dimensional supersymmetric gauge theory is written in terms of $\mathcal{N}=1$, $D=4$ superfields. The theory is dimensionally reduced over six-dimensional coset spaces. We find that the resulting four-dimensional theory is either a softly broken $\mathcal{N}=1$ supersymmetric gauge theory or a non-supersymmetric gauge theory depending on whether the coset spaces used in the reduction are non-symmetric or symmetric. In both cases examples susceptible to yield realistic models are presented.

Keywords: Space-Time Symmetries, Gauge Symmetry, Supersymmetry Breaking, Superspaces.

Contents

1. Introduction

Higher-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories are known to lead to gauge theories with extended supersymmetries in four dimensions. Well known examples are $\mathcal{N} = 1$ supersymmetric gauge theories in ten and six dimensions leading to $\mathcal{N} = 4$ and $\mathcal{N} = 2$ supersymmetric gauge theories in four dimensions under trivial reduction [1]. Four-dimensional gauge theories with extended supersymmetries are very interesting frameworks for studying properties of superconformal field theories and dualities [2, 3]. However, they are not known to lead to realistic theories describing physics beyond the Standard Model (SM). On the other hand the SM requires an understanding of the plethora of its free parameters resulting mostly from the ad-hoc introduction of the Higgs and Yukawa sectors in the theory, which could have their origin in a higher-dimensional theory. Indeed various schemes, with the Coset Space Dimensional Reduction (CSDR) [4, 5, 6] being pioneer have suggested that a unification of the gauge and Higgs sectors can be achieved in higher dimensions; the four-dimensional gauge and Higgs fields are simply the surviving components of the gauge fields of a pure gauge theory defined in higher dimensions. In the next step of development of the CSDR scheme, fermions were introduced [7] and then the four-dimensional Yukawa and gauge interactions of fermions found also a unified description in the gauge interactions of the higher-dimensional theory. The last step in this unified description in higher dimensions is to relate the gauge and fermion fields which have been introduced. This can be achieved by demanding that the higher-dimensional gauge theory be $\mathcal{N} = 1$ supersymmetric; the gauge and fermion fields are members of the same vector supermultiplet. A very welcome input in this line of arguments comes from String Theory

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(for instance the heterotic string) which fixes the space-time dimension and the gauge group of the higher-dimensional supersymmetric theory [1, 8]. Among the successes of the CSDR scheme one has to add the possibility of obtaining chiral theories in four dimensions [9], as well as softly broken or non-supersymmetric theories [10].

Motivated by the old work of Marcus, Sagnotti and Siegel (MSS) [11], a recent article [12] which examines the MSS lagrangian in dimensions between five and ten and by ref. [13], where superspace lagrangians with compact extra fifth dimension are considered, we would like to examine here to which extend our findings in refs. [10] can be described in the superfield language. We find that indeed this is possible with the exception of describing the softly supersymmetry breaking gaugino mass term due to the lack, so far, of a superfield formulation of the ten-dimensional supergravity-supersymmetric Yang-Mills (SYM) lagrangian. For the completeness of presentation we bypass this difficulty by choosing the torsion of the non-symmetric coset space such that there is no gaugino mass generated and carrying out the calculations at the origin of the coset space so that the form of the lagrangian is that of a flat space one. Moreover in our construction of explicit examples we present the details of a new calculation leading to the derivation of the potential of the resulting four-dimensional theory after dimensional reduction. Finally we show how the four-dimensional GUTs obtained using this dimensional reduction can potentially become realistic theories.

The present paper is organized as follows. In section 1 we present the $D = 10$, $\mathcal{N} = 1$ lagrangian in terms of $D = 4$, $\mathcal{N} = 1$ superfields. In section 2 we perform the CSDR using four-dimensional $\mathcal{N} = 1$ superfields. In section 3 using the superfield formulation we present a three-generation softly broken supersymmetric example; it specifically concerns the reduction of a $G = E_8$ SYM theory over $\mathrm{SU}(3)/(\mathrm{U}(1)\times\mathrm{U}(1))$. In section 4 we present a three generation non-supersymmetric example. This example is obtained by the reduction of a $G = E_8$ SYM theory over $B = (\mathrm{SU}(3)/\mathrm{SU}(2)\times\mathrm{U}(1)) \times (\mathrm{SU}(2)/\mathrm{U}(1))$ and the details of the four-dimensional potential are presented for the first time. Finally section 5 contains our conclusions.

2. The $D = 10$, $\mathcal{N} = 1$ lagrangian in terms of $D = 4$, $\mathcal{N} = 1$ superfields

In ref. [11] the ten-dimensional, $\mathcal{N} = 1$ SYM lagrangian was written in terms of four-dimensional $\mathcal{N} = 1$ superfields generalizing earlier formulation of the four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory in terms of $\mathcal{N} = 1$ superfields [14, 15]. The lagrangian written in superfield formalism is equivalent to the known lagrangian expressed in components [16], i.e. it reduces to the latter when the auxiliary fields are eliminated via the equations of motion. Then trivial dimensional reduction to four dimensions, leads to $\mathcal{N} = 4$ supersymmetric gauge theory. In the present work we perform a generalized dimensional reduction, namely the CSDR [4]–[7]. We find that the four-dimensional theory obtained in this way can be a softly broken $\mathcal{N} = 1$ supersymmetric gauge theory [10] if the reduction is carried over a coset space S/R which admits an SU(3)-structure [17], a requirement which in turn leads us to the suitable six-dimensional coset spaces. These are the three well known spaces $G_2/\mathrm{SU}(3)$, $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{non-max}$ and $\mathrm{SU}(3)/\mathrm{U}(1)\times\mathrm{U}(1)$.

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It should be stressed that in general in the process of CSDR any sign of the higher-dimensional supersymmetry may be lost [5, 9, 10]. Roughly the reason is the following. We assume a spacetime of the form $M_4 \times S/R$ with M_4 the four-dimensional Minkowski space and S/R a coset space. We reduce the theory in M_4 by imposing a generalized S-invariance in the extra-dimensional dependence of the fields defined on $M_4 \times S/R$, (the details will be presented in section 3). The effect of this procedure is that only the S-invariant terms in the expansion of the fields in terms of harmonics of S/R survive. The reason¡ for obtaining either a non-supersymmetric or a softly broken supersymmetric theory in four dimensions is due to the fact that S-invariance does not commute with supersymmetry. Either S-invariant bosons and fermions belong to different representations, as in the former case, or when they belong to the same representations, as in the latter case, then the interactions introduced in the process of dimensional reduction break softly even the remaining $\mathcal{N} = 1$ supersymmetry.

The $\mathcal{N} = 4$ vector multiplet in ten dimensions decomposes under $\mathcal{N} = 1$ as 3 chiral multiplets Φ_a, $a = 1, 2, 3$ and a vector multiplet V. The superspace lagrangian will be built out of these fields. These will be ordinary $D = 4, \mathcal{N} = 1$ superfields having a dependence on the extra coordinates y^a which can be considered, from the four-dimensional point of view, just as parameters. Equivalently the fields V and Φ_a can be considered as ordinary fields, i.e. a scalar and a vector on the transverse space, depending on the parameters $x, \theta, \bar{\theta}$.

Following ref. [12] we consider the ten-dimensional $\mathcal{N} = 1$ SYM theory, whose lagrangian is given by

$$\mathcal{L} = -\frac{1}{4g^2}\,\mathrm{Tr}(F_{MN}F^{MN}) - \frac{i}{2g^2}\,\mathrm{Tr}(\overline{\lambda}\Gamma^M D_M \lambda)\,. \tag{2.1}$$

In order to write the theory in terms of $D = 4$, $\mathcal{N} = 1$ superfields on a spacetime of the form $M^4 \times B$ we use the fact that the six-dimensional transverse space B admits an almost complex structure which allows us to use complex coordinates, z^a, $a = 1, 2, 3$, at a particular point, which later will be fixed to be the origin of a coset space $B = S/R$. If $y^1, y^2, y^3, y^4, y^5, y^6$ are the coordinates of the six-dimensional transverse space, then we can introduce complex coordinates by defining

$$z^1 = \frac{1}{2}(y^1 + iy^2) \qquad z^2 = \frac{1}{2}(y^3 + iy^4) \qquad z^3 = \frac{1}{2}(y^5 + iy^6)\,. \tag{2.2}$$

The transverse rotational invariance is the SO(6) rotating the $y^1, \cdots, y^6$ into each other. The SU(3) subgroup of the SO(6) $\cong$ SU(4) that rotates the z^a will be useful in constructing invariant actions. Similarly to the coordinates we can use combinations of the real higher-dimensional components of the gauge field to construct the complex lowest components of the three chiral superfields Φ_a, as follows

$$\Phi_a|_{\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}}A_a = \frac{1}{\sqrt{2}}(A_{4+2a} + iA_{3+2a})\,. \tag{2.3}$$

We will use the convention $\bar{z}^a = (z_a)^\dagger$ for the complex coordinates, and $\bar{\Phi}^a = (\Phi_a)^\dagger$ for the chiral fields.

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The ten-dimensional action then takes the form [11, 12]

$$
\begin{aligned}
S_{10} = \int d^{10}x \Bigg\{ \int d^2\theta \, \mathrm{Tr} \left(\frac{1}{4g^2} W^\alpha W_\alpha + \frac{1}{2g^2} \epsilon^{abc} \Phi_a (\partial_b \Phi_c - \frac{1}{2}[\Phi_b, \Phi_c]) \right) + \\
+ \int d^4\theta \, \frac{1}{g^2} \, \mathrm{Tr}((\bar{\nabla}^a + \bar{\Phi}^a) e^{-V} (\nabla_a + \Phi_a) e^V + \bar{\nabla}^a e^{-V} \nabla_a e^V) \Bigg\} + \text{WZW term} \, .
\end{aligned}
\tag{2.4}
$$

The choice to express the vector supermultiplet in terms of $\mathcal{N} = 1$ superfields breaks the original $\mathcal{R} = \mathrm{SU}(4)$ symmetry of the lagrangian down to $\mathrm{SU}(3)$. The WZW term vanishes in the Wess-Zumino gauge. In ref. [11] was proven that this lagrangian is equivalent to the on-shell $D = 10$, $\mathcal{N} = 1$ SYM given in eq. (2.1). When eq. (2.4) is trivially reduced to four dimensions the $D = 4$, $\mathcal{N} = 4$ lagrangian is obtained. The six transverse components of the gauge field become scalars in the adjoint of the ten-dimensional gauge group G. In components it has the form

$$
\mathcal{L} = -\frac{1}{4g^2} \, \mathrm{Tr} \left(F_{\mu\nu} F^{\mu\nu} + 2 D_\mu A_m D^\mu A_m - [A_m, A_n]^2 \right) - \frac{i}{2g^2} \, \mathrm{Tr}(\overline{\lambda} \Gamma^\mu D_\mu \lambda + i \overline{\lambda} \Gamma^m [A_m, \lambda])
\tag{2.5}
$$

with the potential given by

$$
V = -\frac{1}{4g^2} \, \mathrm{Tr} \left([A_m, A_n] \right)^2 ,
\tag{2.6}
$$

as can be found by inspection of eq. (2.5). In the CSDR described below we will show how both the gauge symmetry G is broken to a subgroup H and the $\mathcal{N} = 4$ SUSY is reduced to softly broken $\mathcal{N} = 1$. Moreover the Higgs fields obtained from the higher-dimensional components of the gauge field do not belong in the adjoint representation.

3. CSDR using four-dimensional $\mathcal{N} = 1$ superfields

Let us proceed by recalling the fundamental ideas of the dimensional reduction procedure that will be used. Dimensional reduction is the construction of a lower-dimensional lagrangian starting from higher dimensions. In our construction we will use the symmetries of the extra dimensions. The CSDR is a dimensional reduction scheme, in which the extra dimensions form a coset space S/R and the symmetry used is the group S of isometries of S/R.

Given the above form of the extra coordinates one can construct a lower-dimensional lagrangian by demanding the fields to be symmetric. This means that one could require the fields to be form invariant under the action of the group S on extra coordinates,

$$
\delta_\xi \Phi^i = L_\xi \Phi^i = 0 \, ,
\tag{3.1}
$$

where L_ξ is the Lie derivative with respect to the Killing vectors ξ of the extra dimensional metric. However this condition appears to be too strong when the higher-dimensional lagrangian possesses a symmetry. Then a generalized form of the symmetry condition (3.1) can be

$$
\delta_\xi \Phi^i = L_\xi \Phi^i = U_\xi \Phi^i \, ,
\tag{3.2}
$$

where U_ξ is the symmetry of the lagrangian.

When we apply CSDR in a gauge theory the U_ξ is naturally chosen to be a gauge transformation. Then the original lagrangian becomes independent of the extra coordinates because of the gauge invariance of the theory.

The original spacetime (M^D, g^{MN}), is assumed to be compactified to $M^4 \times S/R$, with S/R a coset space. The original coordinates $\hat{x}^M$ become coordinates of $M^4 \times S/R$, $\hat{x}^M = (x^\mu, y^\alpha)$, where α is a curved index of the coset and a denotes a flat tangent space index. The metric is

$$g^{MN} = \begin{bmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{\alpha\beta} \end{bmatrix}, \tag{3.3}$$

where $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski spacetime metric and $g^{\alpha\beta}$ is the coset space metric.

In order to study the geometry of coset spaces it is useful to divide the generators of S, Q_A in two sets the generators of R, Q_i $(i = 1, \ldots, \dim R)$ and the generators of S/R, Q_a $(a = \dim R + 1 \ldots, \dim S)$. Then the commutation relations for the S generators become

$$\begin{aligned} [Q_i, Q_j] &= f_{ij}^k Q_k \,, \\ [Q_i, Q_a] &= f_{ia}^b Q_b \,, \\ [Q_a, Q_b] &= f_{ab}^i Q_i + f_{ab}^c Q_c \,. \end{aligned} \tag{3.4}$$

The coset space S/R is called symmetric when $f_{ab}^c = 0$. Then the above commutation relations are invariant under the discrete mapping of generators $Q_a \to -Q_a$ [19].

The coordinates y define an element of S, $L(y)$, which is a coset representative. Then the Maurer-Cartan form with values in the Lie algebra of S is defined by

$$V(y) = L^{-1}(y) dL(y) = e_\alpha^A Q_A dy^\alpha \tag{3.5}$$

and obeys the Maurer-Cartan equation,

$$dV + V \wedge V = 0 \,. \tag{3.6}$$

From the relation (3.6), and using standard techniques of differential geometry, we can develop the geometry of coset spaces and compute vielbeins, connections, curvature and torsion. For instance the vielbein and the R-connection can be computed at the origin, $y = 0$; the results being $e_\alpha^a = \delta_\alpha^a$ and $e_\alpha^i = 0$. More generally in the CSDR scheme we can perform the necessary calculations at the origin, $y = 0$ of the coset space S/R due to the transitive action of S on S/R.

The fermion part of the lagrangian (2.1), when reduced according to CSDR rules (see e.g. [5]) becomes

$$L_Y = \frac{i}{2} \overline{\psi} \Gamma^a \nabla_a \psi + \overline{\psi} V \psi \,. \tag{3.7}$$

At $y = 0$ one finds that $\nabla_a = \phi_a$ and therefore the term $\frac{i}{2}\overline{\psi}\Gamma^a \nabla_a \psi$ in eq. (3.7) is exactly the Yukawa term. The last term more explicitly is given by

$$V = \frac{i}{4} \Gamma^a G_{abc} \Sigma^{bc} \,, \tag{3.8}$$

S/R	SO(6) vector	SO(6) spinor
$\mathrm{SO}(7)/\mathrm{SO}(6)$	6	4
$\mathrm{SU}(4)/\mathrm{SU}(3)\times\mathrm{U}(1)$	$3_{-2}+\overline{3}_2$	1_3+3_{-1}
$\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\mathrm{max}}$	$3_{-2}+3_2$	1_3+3_{-1}
$\mathrm{SU}(3)\times\mathrm{SU}(2)/\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$	$1_{0,2a}+1_{0,-2a}$ $+2_{b,0}+2_{-b,0}$	$1_{b,-a}+1_{-b,-a}$ $+2_{0,a}$
$\mathrm{Sp}(4)\times\mathrm{SU}(2)/\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$(1,1)_2+(1,1)_{-2}$ $+(2,2)_0$	$(2,1)_1+(1,2)_{-1}$
$(\mathrm{SU}(2)/\mathrm{U}(1))^3$	$(2a,0,0)+(-2a,0,0)$ $+(0,2b,0)+(0,-2b,0)$ $(0,0,2c)+(0,0,-2c)$	$(a,b,c)+(-a,-b,c)$ $+(-a,b,-c)+(a,-b,-c)$

Table 1: Six-dimensional symmetric cosets with rank$S=$ rank$R$.

where Γ^a are the gamma matrices in six dimensions, $\Sigma^{bc}=1/2(\Gamma^a\Gamma^b-\Gamma^b\Gamma^a)$ and $G_{abc}=(1+k)f_{abc}$, with k a parameter which controls the torsion and is the term from which the gaugino acquires mass. Clearly the term (3.8) can be put equal to zero by the choice $k=-1$. Then the total action,

$$S=\int d^4xd^dy\sqrt{-g}\left[-\frac{1}{4}\,\mathrm{Tr}\left(F_{MN}F_{K\Lambda}\right)g^{MK}g^{N\Lambda}+\frac{i}{2}\overline{\psi}\Gamma^M D_M\psi\right], \qquad (3.9)$$

can be written as [11]

$$S_{10}=\int d^{10}x\left\{\int d^2\theta\,\mathrm{Tr}\left(\frac{1}{4g^2}W^\alpha W_\alpha+\frac{1}{2g^2}\epsilon^{abc}\Phi_a(\partial_b\Phi_c-\frac{1}{2}[\Phi_b,\Phi_c])\right)+\right.$$
$$\left.+\int d^4\theta\,\frac{1}{g^2}\,\mathrm{Tr}((\bar\partial^a+\bar\Phi^a)e^{-V}(\partial_a+\Phi_a)e^V+\bar\partial^a e^{-V}\partial_a e^V)\right\}. \qquad (3.10)$$

The action (3.10) is equivalent to the ten-dimensional SYM on flat space $M_4\times T_{y=0}(S/R)$. The action for the ten-dimensional $\mathcal{N}=1$ supergravity-SYM theory in terms of four-dimensional superfields is not known yet. This would be the desired form to be used in order to demonstrate the occurrence of a geometrical mass term for the gaugino in the superfield language.

Note that in order to reduce the lagrangian (3.10) over a coset space S/R we need complex coordinates z^a, $z^{\bar a}$ at least at the point $y=0$. Also recall that the subgroup of SU(4) which leaves invariant one supersymmetry is SU(3). If we assume that the extra dimensions form a compact space, then the original SO(6) structure group of the frame bundle should be reducible to SU(3) [1, 8]. In order to determine the possible $\mathcal{N}=1$ reductions we have to select among the available coset spaces those which admit an SU(3)-structure. Then the original form of the lagrangian with the three chiral superfields will be kept in the process of dimensional reduction. For instance the chiral superfields correspond to $(1,0)$ 1-forms, the complex conjugate antichiral superfield corresponds to $(0,1)$ 1-forms. Then the $(1,0)+(0,1)$-form $\Phi=\Phi_a dz^a+\Phi_{\bar a}d\bar z^{\bar a}$ corresponds to the decomposition $6=3+\bar 3$ in the embedding $\mathrm{SO}(6)\supset\mathrm{SU}(3)$ (See tables 1 and 2). Thus we are led to the known non-symmetric coset spaces $G_2/\mathrm{SU}(3)$, $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\mathrm{non.max}}$, $\mathrm{SU}(3)/\mathrm{U}(1)\times\mathrm{U}(1)$. The

S/R	SO(6) vector	SO(6) spinor
$G_2/\,\mathrm{SU}(3)$	$3+\overline{3}$	$1+3$
$\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\mathrm{non-max}}$	$1_2+1_{-2}+2_1+2_{-1}$	$1_0+1_2+2_{-1}$
$\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$	$(a,c)+(b,d)+(a+b,c+d)$ $+(-a,-c)+(-b,-d)$ $+(-a-b,-c-d)$	$(0,0)+(a,c)+(b,d)$ $+(-a-b,-c-d)$

Table 2: Six-dimensional non-symmetric cosets with rank$S$ = rank$R$.

SU(3)-structure is defined through the invariant tensor f_{abc}. An SU(3)-structure cannot be defined for the symmetric coset spaces given that such an invariant tensor does not exist.

We could have reached to the same conclusions if we have examined the constraints in component form. In that case we would find that if the cosets are symmetric then the components of the original superfields belong to different representations of the gauge group and therefore no sign of the original supersymmetry would be left in four dimensions. In the case of non-symmetric cosets we would find a softly broken supersymmetric theory in four dimensions, i.e. although the reduction preserves a supersymmetric spectrum, the interactions that are introduced through the constraints break the supersymmetry softly.

In the superfield formalism the four-dimensional superfields depend on extra dimensions. From the four-dimensional point of view the extra dimensions can be considered as continuous parameters. From the internal space point of view the vector superfield is just a scalar function of the coordinates $V(y)$, while the three chiral superfields constitute a 1-form $\Phi_a(y)$. This fact makes easy the formulation of the CSDR scheme using four-dimensional superfields in the case of non-symmetric coset spaces. Again we demand that the superfields should be invariant up to a gauge transformation when S acts on S/R. We denote by ξ^α_A, $A=1,\ldots,\dim S$, the Killing vectors which generate the isometries S of S/R and by W_A we denote the compensating gauge tranformation which corresponds to ξ_A. We also define the infinitesimal motion

$$\delta_A \equiv L_{\xi_A}\,. \tag{3.11}$$

Then the condition that an infinitesimal motion is compensated by a gauge transformation takes the following specific form when applied to the scalar V and to the 1-form Φ_a (both belonging to the adjoint representation of the multidimensional gauge group G)

$$\delta_A V = \xi^\alpha_A \partial_\alpha V = [W_A, V]\,, \tag{3.12}$$

$$\begin{aligned}\delta_A \Phi_\alpha &= \xi^\beta_A \partial_\beta \Phi_\alpha + \partial_\alpha \xi^\beta_A \Phi_\beta \\ &= \partial_\alpha W_A - [W_A, \Phi_\alpha]\,.\end{aligned} \tag{3.13}$$

The W_A generally depend on the internal coordinates y. The conditions (3.12) and (3.13) must be covariant when $\Phi_\alpha(y)$ and $V(y)$ transform under a gauge tranfomation. We conclude that W transforms as

$$\widetilde{W}_A = gW_Ag^{-1} + (\delta_A g)g^{-1}. \tag{3.14}$$

The variations δ_A, satisfy the condition $[\delta_A, \delta_B] = f_{AB}^{\ C}\delta_C$. This leads to another consistency condition on W, namely

$$\xi_A^\alpha \partial_\alpha W_B - \xi_B^\alpha \partial_\alpha W_A + [W_A, W_B] = f_{AB}^{\ C} W_C \,. \tag{3.15}$$

We note that at the origin, $y = 0$ the Killing vectors take the form

$$\xi_a^\alpha = \delta_a^\alpha \,, \qquad \xi_i^\alpha = 0 \,, \tag{3.16}$$

which mark out the point $y = 0$ as the most suitable point for performing calculations. From eq. (3.14) we conclude that W_a can always become zero, while this is not true for W_i. Therefore eqs (3.12), (3.13) and (3.14) provide us the freedom to perform our calculations at $y = 0$ and to put $W_a = 0$. Under these assumptions eq. (3.15) gives

$$\partial_a W_b - \partial_b W_a = f_{abi} W^i \,, \tag{3.17}$$

$$\partial_a W_i = 0 \,, \tag{3.18}$$

$$[W_i, W_j] = f_{ij}^{\ \ k} W_k \,. \tag{3.19}$$

The W_i's are constant and equal to the generators of the R-subgroup of G. The W_i will be denoted by Φ_i, and therefore if $\hat{a} = (1, \ldots, \dim\ S)$, we will have $\Phi_{\hat{a}} = (\Phi_a, \Phi_i)$ with $a = (1, \ldots, \dim S/R)$.

Next we will analyze the constraints (3.12) and (3.13). From eq. (3.12) we obtain at $y = 0$

$$\partial_a V = 0 \,, \qquad [W_i, V] = 0 \,. \tag{3.20}$$

Eq. (3.20) shows that the four-dimensional vector superfield is independent of the coordinates of the coset space and belongs to the adjoint representation of the reduced gauge group H which is the subgroup of G commuting with R, i.e. H is the centralizer of the image of R in G.

In order to reduce the 1-form Φ_a we use eq. (3.13) and obtain at $y = 0$

$$\partial_a \Phi_b - \frac{1}{2} f^{\bar{c}}{}_{ab} \Phi_{\bar{c}} = \partial_b \Phi_a - \frac{1}{2} f^{\bar{c}}{}_{ba} \Phi_{\bar{c}} \,, \tag{3.21}$$

and

$$[W_i, \Phi_a] = f_{ia}{}^{c} \Phi_c \,. \tag{3.22}$$

Eq. (3.21) makes the chiral superfields non-propagating in the extra dimensions. Eq. (3.22) can be analyzed using Schur's lemma [4, 5] providing the rules that determine the chiral superfields surviving from the reduction of Φ_a. The rules are the following. First we embed R in S and decompose the adjoint representation of S under R,

$$\begin{aligned} S &\supset R \\ \mathrm{adj}\, S &= \mathrm{adj}\, R + \sum s_i . \end{aligned} \tag{3.23}$$

Then we embed R in G and decompose the adjoint of G under $R \times H$,

$$\begin{aligned} G &\supset R_G \times H \\ \mathrm{adj}\, G &= (\mathrm{adj}\, R, 1) + (1, \mathrm{adj}\, H) + \sum (r_i, h_i) \,. \end{aligned} \tag{3.24}$$

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The rule is that when

$$s_i = r_i \,,$$

i.e. when there exist two identical representations of R in the decompositions (3.23) and (3.24), there is one h_i multiplet of chiral superfields surviving in four dimensions. If Q_{ax} are the generators of G belonging to the $\sum(r_i, h_i)$ part of the decomposition, i.e. a is an r_i index and x is an h_i index, then the unconstrained superfields are B^x with

$$\Phi_a = B^x Q_{ax} \,. \tag{3.25}$$

Next we reduce the action (3.10). The superpotential $\mathcal{W}$ is

$$\mathcal{W} = \epsilon^{abc}\Phi_a \left(\partial_b \Phi_c - \frac{1}{2}[\Phi_b, \Phi_c]\right) \tag{3.26}$$

and can be written in a more transparent form in order to use the CSDR constraints, as

$$\mathcal{W} = \frac{1}{2}\epsilon^{abc}\Phi_a(\partial_b\Phi_c - \partial_c\Phi_b) - \frac{1}{2}\epsilon^{abc}\Phi_a[\Phi_b, \Phi_c] \,. \tag{3.27}$$

Then using the constraints of CSDR we obtain

$$\partial_b \Phi_c = \partial_b W_c + \frac{1}{2} f_{\ bcd}\bar{\Phi}^d \,, \tag{3.28}$$

and antisymmetrizing the indices bc we obtain

$$\partial_b\Phi_c - \partial_c\Phi_b = \partial_c W_b - \partial_b W_c + f_{\ bcd}\bar{\Phi}^d \,. \tag{3.29}$$

Next due to the constraint (3.17) we obtain,

$$\partial_c W_b - \partial_b W_b = f_{\ bci}\bar{\Phi}^i \tag{3.30}$$

and the superpotential (3.27) takes the form

$$\mathcal{W} = \frac{1}{2}\,\mathrm{Tr}\left(\epsilon^{abc}\Phi_a(f_{\ bc\hat{d}}\bar{\Phi}^{\hat{d}} - [\Phi_b, \Phi_c])\right) \,. \tag{3.31}$$

Setting $(M)^a_{\hat{b}} \equiv \epsilon^{abc} f_{\hat{b}bc}$ we see that a non-holomorphic term

$$(M)^a_b \Phi_a \bar{\Phi}^b \tag{3.32}$$

has occurred in $\mathcal{W}$, in the process of dimensional reduction, which breaks supersymmetry and provides the four-dimensional lagrangian with scalar mass soft terms. On the other hand, we identify as superpotential W of the four-dimensional theory the holomorphic part of the original superpotential, $\mathcal{W}$

$$W = -\frac{1}{2}\,\mathrm{Tr}\left(\epsilon^{abc}\Phi_a[\Phi_b, \Phi_c]\right) \,. \tag{3.33}$$

To determine the contribution of the term (3.32) we should compute $M\bar{\Phi}\Phi|_{\theta^2}$. Using the form of Φ in components,

$$\Phi(x, y) = \phi(x, y) + \theta^2 F(x, y) + \cdots \tag{3.34}$$

and

$$\bar{\Phi} = \phi^*(x,y) + \bar{\theta}^2 F^*(x,y) + \cdots\,, \tag{3.35}$$

we conclude that

$$M\bar{\Phi}\Phi|_{\theta^2} = MF\phi^*\,.$$

From the equations of motion for F we obtain

$$F^* = -\left(\frac{\partial W}{\partial \phi}\phi + M\phi^*\right). \tag{3.36}$$

Then substituting F^*, as given in eq. (3.36), in the lagrangian expressed in components and specifically in the terms

$$\frac{1}{2}F^*F + \frac{1}{2}\frac{\partial \overline{W}}{\partial \phi^*}F^* + MF^*\phi$$

we obtain finally the familiar supersymmetric F-terms plus terms which break supersymmetry softly, i.e.

$$\left|\frac{\partial W}{\partial \phi}\right|^2 + M\frac{\partial W}{\partial \phi}\phi + M^2\phi^2\,. \tag{3.37}$$

Next we examine the interaction term

$$(\bar{\partial}^a + \bar{\Phi}^a)e^{-V}(\partial_a + \Phi_a)e^V + \bar{\partial}^a e^{-V}\partial_a e^V\,. \tag{3.38}$$

We shall show how the usual kinetic term of $\mathcal{N}=1$ theories

$$\mathrm{Tr}\left(\bar{\Phi}e^{-V}\Phi\right) \tag{3.39}$$

is obtained. In order to reduce the kinetic term we consider the relevant part of eq. (3.10)

$$\mathrm{Tr}\left(\bar{\Phi}^a e^{-V}\Phi_a e^V\right) \tag{3.40}$$

and write the components in the Wess-Zumino gauge

$$\bar{\Phi}^a\left(1 - V + \frac{1}{2}V^2\right)\Phi_a\left(1 + V + \frac{1}{2}V^2\right), \tag{3.41}$$

where both the vector and chiral superfields belong to the adjoint representation of G

$$\Phi_a = \Phi_a^\alpha T^\alpha\,, \qquad V = V^\alpha T^\alpha\,. \tag{3.42}$$

Then, taking into account the commutator $[T^\alpha, T^\beta] = g^{\alpha\beta\gamma}T^\gamma$, we find

$$\bar{\Phi}^a\Phi_a - g_{\alpha\beta\gamma}\bar{\Phi}^{a\alpha}V^\beta\Phi_a^\gamma + \frac{1}{2}g^\epsilon_{\alpha\beta}g_{\epsilon\gamma\delta}\bar{\Phi}^{a\alpha}V^\beta V^\gamma\Phi_a^\delta\,. \tag{3.43}$$

When the representations of G are reduced in those of H, the structure constants $g_{\alpha\beta\gamma}$ of G are reduced accordingly to the structure constants of H. The kinetic term thus obtained is the familiar one,

$$\mathrm{Tr}\,\bar{\Phi}(e^{-V})\Phi\,. \tag{3.44}$$

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Let us see this point in some more detail. Initially we had chiral superfields Φ_a^α with $a = 1, 2, 3$ and $\alpha = 1 \ldots dim\ G$ an index in the adjoint representation of the higher-dimensional gauge group G. Through the reduction procedure we obtain $\Phi_a^\alpha \to \Phi_{\hat{a}}^x$ with $\hat{a}$ counting the number of the surviving generations; it depends on the non-symmetric coset over which we reduce the theory. Specifically $\hat{a} = 1$ if we reduce the theory over $G_2/\,\mathrm{SU}(3)$, $\hat{a} = 1, 2$ if we reduce the theory over $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\mathrm{non.max}}$ and $\hat{a} = 1, 2, 3$ if we reduce the theory over $\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$ [10]. The index x denotes the H-representation in which the surviving chiral superfields belong. We will further denote the adjoint of the reduced gauge group H with an index i. Therefore the reduction can be represented as

$$\Phi^{a\alpha}(f^\gamma)_\alpha^\beta V^\gamma \Phi_{a\beta} \to \Phi^{\hat{a}x}(f^i)_x^y V^i \Phi_{\hat{a}y}\,.$$

A final remark concerning the dimensional reduction described in the present section is the following. According to the rules of the reduction obtained from the constraints we have to embed R in G. In turn the embedding determines the four-dimensional gauge group H as being the centralizer of R in G, i.e. $H = C_G(R)$. A second role of this embedding, or more accurately of identifying R with a subgroup of G, is that it provides a non-trivial background configuration which could result in obtaining chiral fermions in four dimensions. The index of the Dirac operator is a topological invariant depending only on the coset space and the background gauge fields [5, 19].

Usually one considers a simple background configuration and the chirality index is related with the Euler characteristic of a given coset space [18, 19, 5] (see however [20]). The point we would like to stress here is that, in principle, one could consider background configurations with non-trivial winding number. This offers the possibility to obtain more fermion families in four dimensions, a fact particularly useful when one considers reduction over coset spaces which are multiply connected.

Multiply connected spaces in the present framework can result by moding out a freely acting discrete group from a coset space and can be used to break the gauge group using the so called Wilson flux or Hosotani mechanism [21, 22, 5]. Therefore, what we should do in order to use the possibility offered by considering a background configuration with winding number n is to perform a large gauge transformation, which will take us from one vacuum to another with different winding number, followed by a small gauge transformation which will compensate the change of field in higher dimensions as was described in the present section. The first gauge transformation will provide the necessary chiral fermion multiplicity and the latter the geometrical breaking of the gauge group G down to H in four dimensions.

4. A three generation softly broken supesymmetric example: reduction of the $G = E_8$ over $\mathrm{SU}(3)/(\mathrm{U}(1)\times\mathrm{U}(1))$.

In this section, making use of the reduction in the superfield formalism developed in section 3, we present explicitly the reduction of a $G = E_8$ SYM theory over the coset $\mathrm{SU}(3)/(\mathrm{U}(1)\times\mathrm{U}(1))$. This reduction has been carried out in components in ref. [10]. The decomposition to be used is

$$E_8 \supset \mathrm{U}(1)\times\mathrm{U}(1)\times E_6\,.$$

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The 248 representation of E_8 is decomposed under $\mathrm{U}(1) \times \mathrm{U}(1) \times E_6$ as

$$\begin{aligned}248 = \; & 1_{(0,0)} + 1_{(0,0)} + 1_{(3,\frac{1}{2})} + 1_{(-3,\frac{1}{2})} + 1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-\frac{1}{2})} + 1_{(3,-\frac{1}{2})} + \\ & + 78_{(0,0)} + 27_{(3,\frac{1}{2})} + 27_{(-3,\frac{1}{2})} + 27_{(0,-1)} + \overline{27}_{(-3,-\frac{1}{2})} + \overline{27}_{(3,-\frac{1}{2})} + \overline{27}_{(0,1)} \, . \end{aligned} \tag{4.1}$$

In the present case R is chosen to be identified with the $\mathrm{U}(1) \times \mathrm{U}(1)$ of the decomposition (4.1). Therefore the resulting four-dimensional gauge group, according to the rule stated in eq. (3.20), is

$$H = C_{E_8}(\mathrm{U}(1) \times \mathrm{U}(1)) = \mathrm{U}(1) \times \mathrm{U}(1) \times E_6 \, ,$$

i.e. we find that the surviving fields in four dimensions are three $\mathcal{N} = 1$ vector multiplets $V^\alpha, V_{(1)}, V_{(2)}$, (where α is an E_6, 78 index and the other two refer to the two $\mathrm{U}(1)'s$) containing the gauge fields of $\mathrm{U}(1) \times \mathrm{U}(1) \times E_6$, i.e. the original vector superfield V^A (where A a 248 index of E_8) gives $V^\alpha, V_{(1)}, V_{(2)}$.

The $R = \mathrm{U}(1) \times \mathrm{U}(1)$ content of $\mathrm{SU}(3)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ vector is according to table 2,

$$\left(3, \frac{1}{2}\right) + \left(-3, \frac{1}{2}\right) + (0,-1) + \left(-3, -\frac{1}{2}\right) + \left(3, -\frac{1}{2}\right) + (0,1) \, .$$

Next we apply the solutions given in eqs. (3.23), (3.24) on the surviving chiral superfields of the constraint (3.22) and we find the following answer. The matter content of the four-dimensional theory consists of three $\mathcal{N} = 1$ chiral multiplets (A^i, B^i, C^i) with i an E_6, 27 index and three $\mathcal{N} = 1$ chiral multiplets (A, B, C) which are E_6 singlets and carry $\mathrm{U}(1) \times \mathrm{U}(1)$ charges i.e. the original chiral superfields Φ^A_a (where A is a 248 index of E_8 as before and $a = 1, 2, 3$) give (A^i, B^i, C^i A, B, C).

To understand the above result in detail we examine further the decomposition of the adjoint of the specific $S = \mathrm{SU}(3)$ under $R = \mathrm{U}(1) \times \mathrm{U}(1)$, i.e.

$$\begin{aligned} & \mathrm{SU}(3) \supset \mathrm{U}(1) \times \mathrm{U}(1) \\ & \qquad 8 = (0,0) + (0,0) + \left(3, \frac{1}{2}\right) + \left(-3, \frac{1}{2}\right) + \\ & \qquad\qquad + (0,-1) + \left(-3, -\frac{1}{2}\right) + \left(3, -\frac{1}{2}\right) + (0,1) \, . \end{aligned} \tag{4.2}$$

Then according to the decomposition (4.2) the generators of SU(3) can be grouped as

$$Q_{\mathrm{SU}(3)} = \{Q_0, Q'_0, Q_1, Q_2, Q_3, Q^1, Q^2, Q^3\} \, . \tag{4.3}$$

(The non trivial commutator relations of SU(3) generators (4.3) are given in table 3). The decomposition (4.3) suggests to introduce the following notation for the four-dimensional constrained chiral superfields

$$(\Phi_1, \Phi^1, \Phi_2, \Phi^2, \Phi_3, \Phi^3) \, , \tag{4.4}$$

with $\Phi^a \equiv \bar{\Phi}^a = (\Phi_a)^\dagger$, $a = 1, 2, 3$.

$[Q_1, Q_0] = \sqrt{3} Q_1$	$[Q_1, Q_0'] = Q_1$	$[Q_2, Q_0] = -\sqrt{3} Q_2$
$[Q_2, Q_0'] = Q_2$	$[Q_3, Q_0] = 0$	$[Q_3, Q_0'] = -2 Q_3$
$[Q_1, Q^1] = -\sqrt{3} Q_0 - Q_0'$	$[Q_2, Q^2] = \sqrt{3} Q_0 - Q_0'$	$[Q_3, Q^3] = 2 Q_0'$
$[Q_1, Q_2] = \sqrt{2} Q^3$	$[Q_2, Q_3] = \sqrt{2} Q^1$	$[Q_3, Q_1] = \sqrt{2} Q^2$

Table 3: Non-trivial commutation relations of $SU(3)$ according to the decomposition given in eq.(4.3).

$[Q_1, Q_0] = \sqrt{30} Q_1$	$[Q_1, Q_0'] = \sqrt{10} Q_1$	$[Q_2, Q_0] = -\sqrt{30} Q_2$
$[Q_2, Q_0'] = \sqrt{10} Q_2$	$[Q_3, Q_0] = 0$	$[Q_3, Q_0'] = -2\sqrt{10} Q_3$
$[Q_1, Q^1] = -\sqrt{30} Q_0 - \sqrt{10} Q_0'$	$[Q_2, Q^2] = \sqrt{30} Q_0 - \sqrt{10} Q_0'$	$[Q_3, Q^3] = 2\sqrt{10} Q_0'$
$[Q_1, Q_2] = \sqrt{20} Q^3$	$[Q_2, Q_3] = \sqrt{20} Q^1$	$[Q_3, Q_1] = \sqrt{20} Q^2$
$[Q_{1i}, Q_0] = \sqrt{30} Q_{1i}$	$[Q_{1i}, Q_0'] = \sqrt{10} Q_{1i}$	$[Q_{2i}, Q_0] = -\sqrt{30} Q_{2i}$
$[Q_{2i}, Q_0'] = \sqrt{10} Q_{2i}$	$[Q_{3i}, Q_0] = 0$	$[Q_{3i}, Q_0'] = -2\sqrt{10} Q_{3i}$
$[Q_{1i}, Q_{2j}] = \sqrt{20} d_{ijk} Q^{3k}$	$[Q_{2i}, Q_{3j}] = \sqrt{20} d_{ijk} Q^{1k}$	$[Q_{3i}, Q_{1j}] = \sqrt{20} d_{ijk} Q^{2k}$
$[Q^\alpha, Q^\beta] = 2ig^{\alpha\beta\gamma} Q^\gamma$	$[Q^\alpha, Q_0] = 0$	$[Q^\alpha, Q_0'] = 0$
$[Q^\alpha, Q_{1i}] = -(G^\alpha)_i^j Q_{1j}$	$[Q^\alpha, Q_{2i}] = -(G^\alpha)_i^j Q_{2j}$	$[Q^\alpha, Q_{3i}] = -(G^\alpha)_i^j Q_{3j}$

Table 4: Non-trivial commutation relations of E_8 according to the decomposition given by eq. (4.5).

$$\begin{aligned}
[Q_{1i}, Q^{1j}] &= -\frac{1}{6}(G^\alpha)_i^j Q^\alpha - \sqrt{30}\delta_i^j Q_0 - \sqrt{10}\delta_i^j Q_0' \\
[Q_{2i}, Q^{2j}] &= -\frac{1}{6}(G^\alpha)_i^j Q^\alpha + \sqrt{30}\delta_i^j Q_0 - \sqrt{10}\delta_i^j Q_0' \\
[Q_{3i}, Q^{3j}] &= -\frac{1}{6}(G^\alpha)_i^j Q^\alpha + 2\sqrt{10}\delta_i^j Q_0'
\end{aligned}$$

Table 5: Further non-trivial commutation relations of E_8 according to the decomposition given in eq. (4.5).

Next we examine the commutation relations of E_8 under the decomposition (4.1). Under this decomposition the generators of E_8 can be grouped as

$$Q_{E_8} = \{Q_0, Q_0', Q_1, Q_2, Q_3, Q^1, Q^2, Q^3, Q^\alpha, Q_{1i}, Q_{2i}, Q_{3i}, Q^{1i}, Q^{2i}, Q^{3i}\}\,, \qquad (4.5)$$

where, $\alpha = 1, \ldots, 78$ and $i = 1, \ldots, 27$. The non-trivial commutation relations of the E_8 generators (4.5) are given in tables 4 and 5. Now the constraints (3.22) of the reduction scheme on the superfields given in (4.4) can be expressed as

$$\begin{aligned}
[\Phi_1, \Lambda Q_0] &= \sqrt{3}\Phi_1\,, & [\Phi_1, \Lambda' Q_0'] &= \Phi_1\,, \\
[\Phi_2, \Lambda Q_0] &= -\sqrt{3}\Phi_2\,, & [\Phi_2, \Lambda' Q_0'] &= \Phi_2\,, \\
[\Phi_3, \Lambda Q_0] &= 0\,, & [\Phi_3, \Lambda' Q_0'] &= -2\Phi_3\,. \qquad (4.6)
\end{aligned}$$

Then the solutions of the constraints (4.6) in terms of the genuine four-dimensional chiral superfields and of the E_8 generators (4.5), corresponding to the embedding (4.1) of $R = \mathrm{U}(1) \times \mathrm{U}(1)$ in the E_8, are

$$\Phi_1 = R_1 A^i Q_{1i} + R_1 A Q_1\,,$$

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$$
\begin{aligned}
\Phi_2 &= R_2 B^i Q_{2i} + R_2 B Q_2\,, \\
\Phi_3 &= R_3 C^i Q_{3i} + R_3 C Q_3\,,
\end{aligned}
\tag{4.7}
$$

with $\Lambda = \Lambda' = 1/\sqrt{10}$. Moreover the unconstrained chiral superfields transform under $E_6 \times \mathrm{U}(1) \times \mathrm{U}(1)$ as

$$
\begin{aligned}
A_i &\sim 27_{(3,\frac{1}{2})}\,, & A &\sim 1_{(3,\frac{1}{2})}\,, \\
B_i &\sim 27_{(-3,\frac{1}{2})}\,, & B &\sim 1_{(-3,\frac{1}{2})}\,, \\
C_i &\sim 27_{(0,-1)}\,, & C &\sim 1_{(0,-1)}\,.
\end{aligned}
\tag{4.8}
$$

In terms of the unconstrained superfields the superpotential (3.33) takes the form

$$
W(A^i, B^j, C^k, A, B, C) = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC\,. \tag{4.9}
$$

In addition the trilinear and mass terms which break supersymmetry softly can be read from the two last terms in eq. (3.37) and are given by

$$
\begin{aligned}
\mathcal{L}_{\mathrm{scalar\,SSB}} = & \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\alpha^i\alpha_i + \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\overline{\alpha}\alpha + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\beta^i\beta_i + \\
& + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\overline{\beta}\beta + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\gamma^i\gamma_i + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\overline{\gamma}\gamma + \\
& + \Bigg[\sqrt{2}\,80\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right) d_{ijk}\alpha^i\beta^j\gamma^k + \\
& \quad + \sqrt{2}80\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right)\alpha\beta\gamma + \mathrm{h.c}\Bigg]\,,
\end{aligned}
\tag{4.10}
$$

where α^i etc are the scalar components of the corresponding superfields. The superpotential (4.9), the D-terms and the scalar soft supersymmetry terms given in eq. (4.10) provide the four-dimensional theory with a potential as follows

$$
\begin{aligned}
V(\alpha^i, \alpha, \beta^i, \beta, \gamma^i, \gamma) = \ & \mathrm{const.} + \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\alpha^i\alpha_i + \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\overline{\alpha}\alpha + \\
& + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\beta^i\beta_i + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\overline{\beta}\beta + \\
& + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\gamma^i\gamma_i + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\overline{\gamma}\gamma + \\
& + \Bigg[\sqrt{2}80\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right) d_{ijk}\alpha^i\beta^j\gamma^k + \\
& \quad + \sqrt{2}\,80\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right)\alpha\beta\gamma + \mathrm{h.c}\Bigg] + \\
& + \frac{1}{6}\left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j\right)^2 + \\
& + \frac{10}{6}\left(\alpha^i (3\delta_i^j)\alpha_j + \overline{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \overline{\beta}(-3)\beta\right)^2 + \\
& + \frac{40}{6}\left(\alpha^i\left(\frac{1}{2}\delta_i^j\right)\alpha_j + \overline{\alpha}\left(\frac{1}{2}\right)\alpha + \beta^i\left(\frac{1}{2}\delta_i^j\right)\beta_j + \overline{\beta}\left(\frac{1}{2}\right)\beta + \right.
\end{aligned}
$$

$$+\gamma^i(-1\delta_i^j)\gamma_j+\overline{\gamma}(-1)\gamma\Big)^2+40\alpha^i\beta^j d_{ijk}d^{klm}\alpha_l\beta_m+$$
$$+40\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m+40\alpha^i\gamma^j d_{ijk}d^{klm}\alpha_l\gamma_m+$$
$$+40(\overline{\alpha}\overline{\beta})(\alpha\beta)+40(\overline{\beta}\overline{\gamma})(\beta\gamma)+40(\overline{\gamma}\overline{\alpha})(\gamma\alpha)\,. \tag{4.11}$$

Note that the potential (4.11) belongs to the case that S has an image in G. Here $S = \mathrm{SU}(3)$ has an image in $G = E_8$ and therefore we conclude that the minimum of the potential is zero and the four-dimensional gauge group is E_6 [5]. To break this gauge group further we should employ the Wilson flux mechanism and study the potential (4.11). Given that the Euler number of the coset space $\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$ is 6, therefore leading to three families in four-dimensions, the use of Wilson flux mechanism requires the introduction of a background configuration with non-trivial winding number appropriate to keep unchanged the number of families. We plan to discuss the details and the phenomenological consequences of such construction in a forthcoming publication.

The normalization of generators in the above table is

$$\mathrm{Tr}(Q_0Q_0)=\mathrm{Tr}(Q_0'Q_0')=\mathrm{Tr}(Q_1Q^1)=\mathrm{Tr}(Q_2Q^2)=\mathrm{Tr}(Q_3Q^3)=2\,.$$

The normalization of the generators in tables 4, 5 is

$$\begin{aligned}\mathrm{Tr}(Q_0Q_0)&=\mathrm{Tr}(Q_0'Q_0')=\mathrm{Tr}(Q_1Q^1)=\mathrm{Tr}(Q_2Q^2)=\mathrm{Tr}(Q_3Q^3)=2\,.\\ \mathrm{Tr}(Q_{1i}Q^{1j})&=\mathrm{Tr}(Q_{2i}Q^{2j})=\mathrm{Tr}(Q_{3i}Q^{3j})=2\delta_i^j\,,\\ \mathrm{Tr}(Q^\alpha Q^\beta)&=12\delta^{\alpha\beta}\,.\end{aligned}$$

5. A three generation non-supersymmetric example: reduction of the $G = E_8$ over $B = CP^2 \times S^2$

In the present section we would like to present an interesting three family model in which the original higher-dimensional supersymmetry disappears completely in the process of dimensional reduction. This is the case when we reduce a SYM on symmetric coset spaces. Then the organization of the ten-dimensional lagrangian in terms of four-dimensional superfields is not possible and the reduction must be carried out in the components. The rules are stated separately for bosons and fermions and the surviving four-dimensional fields belong to different representations of the four-dimensional gauge group H. Following refs [4, 5, 7, 10] in the next paragraphs we summarize the results.

To find the four-dimensional gauge group we embed R in G

$$\begin{aligned}G&\supset R_G\times H\,,\\ H&=C_G(R_G)\,.\end{aligned}\tag{5.1}$$

Then H, the centralizer of the image of R in G, is the four-dimensional gauge group.

The scalar fields that are obtained from the higher components of the vector field are obtained as follows. First we embed R in S and decompose the adjoint of S under R

$$\begin{aligned}S&\supset R\\ \mathrm{adj}\,S&=\mathrm{adj}\,R+\sum s_i\,.\end{aligned}\tag{5.2}$$

Then we embed R in G and decompose the adjoint of G under $R \times H$,

$$\begin{aligned} G &\supset R_G \times H \\ \operatorname{adj} G &= (\operatorname{adj} R, 1) + (1, \operatorname{adj} H) + \sum (r_i, h_i)\,. \end{aligned} \tag{5.3}$$

The rule is that when

$$s_i = r_i\,,$$

i.e. when we have two identical representation of R in the decompositions (3.23), (3.24) there is an h_i multiplet of scalar fields that survives in four dimensions.

To find the representation of H under which the four-dimensional fermions transform, we have to decompose the representation F of G to which the fermions belong, under $R_G \times H$, i.e.

$$F = \sum (t_i, h_i)\,, \tag{5.4}$$

and the spinor of $\mathrm{SO}(d)$ under R

$$\sigma_d = \sum \sigma_j\,. \tag{5.5}$$

Then for each pair t_i and σ_i, where t_i and σ_i are identical irreducible representations, survives an h_i multiplet of spinor fields in the four-dimensional theory. In our case we have $F = \operatorname{adj} G$.

Next let us discuss a specific model which appears to be of particular interest. Specifically we choose a ten-dimensional SYM theory based on $G = E_8$ which we reduce over the coset space $S/R = CP^2 \times S^2 = (\mathrm{SU}(3)/\,\mathrm{SU}(2) \times \mathrm{U}(1)) \times (\mathrm{SU}(2)/\,\mathrm{U}(1))$. The embedding of $R = \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ in E_8 is given by the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{SO}(10) \\ 248 &= (1,45)_{(0,0)} + (3,1)_{(0,0)} + (1,1)_{(0,0)} + (1,1)_{(0,0)} + (1,1)_{(2,0)} + (1,1)_{(-2,0)} \\ &\quad + (2,1)_{(1,2)} + (2,1)_{(-1,2)} + (2,1)_{(-1,-2)} + (2,1)_{(1,-2)} + (1,10)_{(0,2)} + \\ &\quad + (1,10)_{(0,-2)} + (2,10)_{(1,0)} + (2,10)_{(-1,0)} + (2,16)_{(0,1)} + (1,16)_{(1,-1)} + \\ &\quad + (1,16)_{(-1,-1)} + (2,\overline{16})_{(0,-1)} + (1,\overline{16})_{(-1,1)} + (1,\overline{16})_{(1,1)}\,, \end{aligned} \tag{5.6}$$

where the $R = \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ is identified with the one appearing in the following decomposition of maximal subgroups

$$\mathrm{SO}(6) \supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1) \supset \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$$

and the $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ in E_8 is the maximal subgroup of $\mathrm{SO}(6)$ appearing in the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SO}(6) \times \mathrm{SO}(10) \\ 248 &= (15,1) + (1,45) + (6,10) + (4,16) + (\overline{4},\overline{16})\,. \end{aligned} \tag{5.7}$$

We find that the four-dimensional gauge group is $H = C_{E_8}(\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)) = \mathrm{SO}(10) \times \mathrm{U}(1) \times \mathrm{U}(1)$. The vector and spinor content under R of the specific coset can

$[Q_\rho, Q_\sigma] = 2i\epsilon_{\rho\sigma\tau} Q_\tau$	$[Q_0, Q_{(+)a}] = Q_{(+)a}$
$[Q_\rho, Q_{(+)a}] = (\tau_\rho)_a^b Q_{(+)b}$	$[Q_{(+)a}, Q_{(-)b}] = (\tau_\rho)_{b\ a} Q_\rho + \delta_{a\ b} Q_0$

Table 6: Non-trivial commutation relations of SU(3).

be found in table 1. Choosing $a = b = 1$ we find that the scalar fields of the four-dimensional theory transform as $10_{(0,2)}$, $10_{(0,-2)}$, $10_{(1,0)}$, $10_{(-1,0)}$ under H. Also, we find that the fermions of the four-dimensional theory are the following left-handed multiplets of H: $16_{(-1,-1)}$, $16_{(1,-1)}$, $16_{(0,1)}$.

To determine the potential we examine further the decomposition of the adjoint of $S = \mathrm{SU}(3) \times \mathrm{SU}(2)$ under $R = \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ The specific coset has the direct product structure

$$\frac{\mathrm{SU}(3)}{\mathrm{SU}(2) \times \mathrm{U}(1)} \times \frac{\mathrm{SU}(2)}{\mathrm{U}(1)}$$

and the embeddings are

$$\mathrm{SU}(3) \supset \mathrm{SU}(2) \times \mathrm{U}(1)$$

and

$$\mathrm{SU}(2) \supset \mathrm{U}(1)$$

The relevant decompositions of the adjoint of SU(3) and SU(2) are respectively

$$\begin{aligned} 8 &= 1_0 + 3_0 + 2_1 + 2_{-1} \\ 3 &= (0) + (2) + (-2) \end{aligned} \tag{5.8}$$

The decomposition of the generators of SU(3) and SU(2) are

$$Q_0, Q_\rho, Q_{(+)a}, Q_{(-)a} \tag{5.9}$$

and

$$Q_0', Q_{(+)}, Q_{(-)} \tag{5.10}$$

respectively. The commutation relations of SU(2) are

$$\left[Q_{(+)}, Q_{(-)}\right] = 2Q_0' \,, \qquad \left[Q_0', Q_{(\pm)}\right] = \pm 2Q_{(\pm)} \tag{5.11}$$

with the normalization $\mathrm{Tr}(Q_{(+)}Q_{(-)}) = \mathrm{Tr}(Q_0'^2) = 2$, while the commutation relations of SU(3) are given in table 6.

The normalization of the generators in table 6 is

$$\mathrm{Tr}(Q_\rho Q_\sigma) = 2\delta^{\rho\ \sigma} \,, \qquad \mathrm{Tr}(Q_{(+)a} Q_{(-)b}) = 2\delta_{a\ b} \,, \qquad \mathrm{Tr}(Q_0^2) = 2 \,.$$

According to the decompositions (5.9) and (5.10), we denote the constrained scalar fields by

$$\phi_{(+)}, \phi_{(-)}, \phi_{(+)a}, \phi_{(-)a} \tag{5.12}$$

and then the constraints to be solved become

$$
\begin{aligned}
\left[\phi'_{(0)}, \phi_{(\pm)}\right] &= \pm 2\phi_{(\pm)}\,, \\
\left[\phi_{(0)}, \phi_{(\pm)a}\right] &= \pm \phi_{(\pm)a}\,, \\
\left[\phi_{\rho}, \phi_{(\pm)a}\right] &= \pm (\tau_\rho)_a^{\ b} \phi_{(\pm)b}\,.
\end{aligned}
\tag{5.13}
$$

We find that the potential of any ten-dimensional theory reduced over $CP^2 \times S^2$ written in terms of the fields (5.12) takes the form

$$
\begin{aligned}
V =& \left(\frac{4(\bar{\Lambda}^2 + \Lambda^2)}{R_1^2} + \frac{4\Lambda'^2}{R_2^2}\right) - \frac{1}{2R_1^4}(\tau_\rho)^{ab} \operatorname{Tr} \phi_\rho [\phi_{(+)a}, \phi_{(-)b}] - \frac{1}{2R_1^4}\delta_b^a \operatorname{Tr} \phi_{(0)}[\phi_{(+)a}, \phi_{(-)b}] + \\
&+ \frac{1}{4R_1^4} \operatorname{Tr}\left([\phi_{(+)a}, \phi_{(+)b}][\phi_{(-)a}, \phi_{(-)b}] + [\phi_{(-)a}, \phi_{(+)b}][\phi_{(+)a}, \phi_{(-)b}]\right) - \\
&- \frac{2}{R_2^4} \operatorname{Tr} \phi'_{(0)}[\phi_{(+)}, \phi_{(+)}] + \frac{1}{2R_2^4} \operatorname{Tr}\left([\phi_{(+)}, \phi_{(-)}]^2\right),
\end{aligned}
\tag{5.14}
$$

where R_1 and R_2 are the coset space radii corresponding to the factors CP^2 and S^2 respectively.

To proceed in determining the four-dimensional potential of our specific example with $G = E_8$, we use the embedding (5.6) of $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ in E_8 and divide its generators accordingly,

$$
\begin{aligned}
Q_{E_8} = \Big\{ & G^{\alpha\beta},\ G^\rho,\ G,\ G',\ G_{(+)},\ G_{(-)}, \\
& K_{(+)a},\ K_{(-)a},\ J_{(+)a},\ J_{(-)a}, \\
& Q_{(+)i},\ Q_{(-)i},\ Q_{(+)ai},\ Q_{(-)ai}, \\
& F_{(+)m},\ \bar{F}^m_{(-)},\ H_{(+)m},\ \bar{H}^m_{(-)},\ F_{(+)am},\ \bar{F}^m_{(-)a} \Big\},
\end{aligned}
\tag{5.15}
$$

with the indices $\alpha\beta, i$ and m belong to the $45, 10$ and 16 of SO(10), while ρ and a to the 3 and 2 of SU(2) respectively. The non-trivial commutation relations of the generators (5.15) are given in tables 6 and 7.

The normalization of the generators in tables 6 and 7 is

$$
\begin{aligned}
\operatorname{Tr} G^{\alpha\beta} G_{\gamma\delta} &= 12\delta^\alpha_\gamma \delta^\beta_\delta\,, \\
\operatorname{Tr} G^{\rho\sigma} &= 2\delta^{\rho\sigma}\,, \\
\operatorname{Tr} G_{(+)} G_{(-)} &= \operatorname{Tr} G^2 = \operatorname{Tr} G'^2 = 2\,, \\
\operatorname{Tr} K_{(+)a} K_{(-)b} &= 2\delta_{ab}\,, \\
\operatorname{Tr} Q_{(+)i} Q_{(-)j} &= 2\delta_{ij}\,, \\
\operatorname{Tr} Q_{(+)ai} Q_{(-)bj} &= 2\delta_{ab}\delta_{ij}\,, \\
\operatorname{Tr} F_{(+)m} \overline{F}^n_{(-)} &= 2\delta^n_m\,, \\
\operatorname{Tr} H_{(+)m} \overline{H}^n_{(-)} &= 2\delta^n_m\,, \\
\operatorname{Tr} F_{(+)am} \overline{F}^n_{(-)b} &= 2\delta_{ab}\delta^n_m\,.
\end{aligned}
$$

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	$[G^{\alpha\beta}, G] = 0$	$[G^{\alpha\beta}, G'] = 0$
$[G^{\alpha\beta}, Q_{(+)i}] = (G^{\alpha\beta})_i^j Q_{(+)j}$	$[G^{\alpha\beta}, Q_{(+)ai}] = (G^{\alpha\beta})_i^j Q_{(+)aj}$	$[G^{\alpha\beta}, F_{(+)m}] = (G^{\alpha\beta})_m^n F_{(+)n}$
$[G^{\alpha\beta}, H_{(+)m}] = (G^{\alpha\beta})_m^n H_{(+)n}$	$[G^{\alpha\beta}, F_{(+)am}] = (G^{\alpha\beta})_m^n F_{(+)an}$	
$[G^\rho, G^\sigma] = 2i\epsilon^{\rho\sigma\tau} G^\tau$	$[G^\rho, G] = 0$	$[G^\rho, G'] = 0$
$[G^\rho, K_{(+)a}] = (\tau^\rho)_a^b K_{(+)b}$	$[G^\rho, J_{(+)a}] = (\tau^\rho)_a^b J_{(+)b}$	$[G^\rho, Q_{(+)ai}] = (\tau^\rho)_a^b Q_{(+)bi}$
$[G^\rho, F_{(+)am}] = (\tau^\rho)_a^b F_{(+)bm}$		
$[Q_{(+)i}, Q_{(+)aj}] = \frac{1}{2}\delta_{ij} K_{(+)a}$	$[Q_{(+)i}, Q_{(-)aj}] = \frac{1}{2}\delta_{ij} J_{(+)a}$	$[Q_{(+)ai}, Q_{(+)bj}] = 0$
$[Q_{(+)i}, F_{(+)m}] = C_{imn} \bar{H}^n_{(-)}$	$[Q_{(+)i}, H_{(+)m}] = -C_{imn} \bar{F}^n_{(-)}$	$[Q_{(+)i}, \bar{F}^m_{(-)b}] = 0$
$[Q_{(+)ai}, H_{(+)m}] = C_{imn} \bar{F}^n_{(-)a}$	$[Q_{(+)ai}, F_{(+)bn}] = -\delta_{ab} C_{imn} \bar{H}^n_{(-)}$	$[Q_{(-)ai}, F_{(+)m}] = C_{imn} \bar{F}^n_{(-)a}$
$[Q_{(-)ai}, F_{(+)bm}] = -\delta_{ab} C_{imn} \bar{F}^n_{(-)}$		
$[F_{(+)m}, H_{(+)n}] = C^i_{mn} Q_{(-)i}$	$[F_{(+)m}, F_{(+)an}] = C^i_{mn} Q_{(+)ai}$	$[H_{(+)m}, F_{(+)an}] = C^i_{mn} Q_{(-)ai}$
$[F_{(+)am}, F_{(+)bn}] = 0$		
$[F_{(+)m}, \bar{H}^n_{(-)}] = \delta^n_m G_{(+)}$	$[F_{(+)m}, \bar{F}^n_{(-)a}] = 0$	$[H_{(+)m}, \bar{F}^n_{(-)}] = \delta^n_m G_{(-)}$
$[H_{(+)m}, \bar{F}^n_{(-)a}] = 0$	$[F_{(+)am}, \bar{F}^n_{(-)}] = 0$	$[F_{(+)am}, \bar{F}^n_{(-)b}] = 0$
$[G^{(')}, G_{(+)}] = 0$	$[G^{(')}, K_{(+)a}] = 3K_{(+)a}$	$[G^{(')}, J_{(+)a}] = 3J_{(+)a}$
$[G^{(')}, Q_{(+)i}] = 2Q_{(+)i}$	$[G^{(')}, Q_{(+)ai}] = Q_{(+)ai}$	$[G^{(')}, F_{(+)m}] = -F_{(+)m}$
$[G^{(')}, H_{(+)m}] = -H_{(+)m}$	$[G^{(')}, F_{(+)am}] = -F_{(+)am}$	
$[G_{(+)}, G_{(-)}] = 0$	$[G_{(+)}, J_{(+)a}] = K_{(+)a}$	$[G_{(+)}, K_{(-)a}] = -J_{(-)a}$
$[G_{(+)}, Q_{(-)ai}] = 0$	$[G_{(+)}, H_{(+)m}] = F_{(+)m}$	$[G_{(+)}, \bar{F}^m_{(-)}] = -\bar{H}^m_{(-)}$
$[K_{(+)a}, J_{(-)b}] = -\delta_{ab} G_{(+)}$		
$[K_{(+)a}, Q_{(-)i}] = \frac{1}{2} Q_{(+)ai}$	$[K_{(+)a}, Q_{(-)bj}] = \frac{1}{2}\delta_{ab} Q_{(+)j}$	$[J_{(+)a}, Q_{(-)j}] = \frac{1}{2} Q_{(-)ai}$
$[J_{(+)a}, Q_{(+)bj}] = \frac{1}{2}\delta_{ab} Q_{(+)j}$		
$[K_{(+)a}, \bar{F}^m_{(-)b}] = 0$	$[K_{(+)a}, H_{(+)m}] = 0$	$[J_{(+)}, F_{(+)m}] = 0$
$[J_{(+)a}, \bar{F}^m_{(-)b}] = 0$	$[G, G'] = 0$	

Table 7: Non-trivial commutation relations of E_8 according the decomposition given by eq. (5.6).

$$
\begin{aligned}
[Q_{(+)i}, Q_{(-)j}] &= \tfrac{1}{6}(G^{\alpha\beta})_{ij} G^{\alpha\beta} + 2\delta_{ij} G + 2\delta_{ij} G' \\
[Q_{(+)ai}, Q_{(-)bj}] &= \tfrac{1}{6}\delta_{ab}(G^{\alpha\beta})_{ij} G^{\alpha\beta} + \delta_{ij}(\tau^\rho)_{ab} G^\rho + \delta_{ij} G + \delta_{ij} G' \\
[F_{(+)m}, \bar{F}^n_{(-)}] &= \tfrac{1}{6}(G^{\alpha\beta})^n_m G^{\alpha\beta} - \delta^n_m G - \delta^n_m G' \\
[H_{(+)m}, \bar{H}^n_{(-)}] &= \tfrac{1}{6}(G^{\alpha\beta})^n_m G^{\alpha\beta} - \delta^n_m G - \delta^n_m G' \\
[F_{(+)am}, \bar{F}^n_{(-)b}] &= \tfrac{1}{6}\delta_{ab}(G^{\alpha\beta})^n_m G^{\alpha\beta} + (\tau^\rho)_{ab}\delta^n_m G^\rho - \delta_{ab}\delta^n_m G - \delta_{ab}\delta^n_m G' \\
[K_{(+)a}, K_{(-)b}] &= (\tau^\rho)_{ab} G^\rho + 3\delta_{ab} G + 3\delta_{ab} G' \\
[J_{(+)a}, J_{(-)b}] &= (\tau^\rho)_{ab} G^\rho + 3\delta_{ab} G + 3\delta_{ab} G' \\
[G^{\alpha\gamma}, G^{\beta\delta}] &= 2i(G^{\alpha\delta}\delta^{\gamma\beta} - G^{\gamma\delta}\delta^{\alpha\beta} + G^{\gamma\beta}\delta^{\alpha\delta})
\end{aligned}
$$

Table 8: Further commutation relations of E_8 according to the dcomposition given by eq. (5.6).

The solution of the constraints (5.13) which will provide us with the genuine four-dimensional scalar fields is given by

$$
\begin{aligned}
\phi'_{(0)} &= \Lambda' G', \phi_{(0)} = \Lambda G, \phi_\rho = \bar{\Lambda} G_\rho\,, \\
\phi_{(+)} &= R_1 \varphi^i Q_{(+)i}, \phi_{(-)} = R_1 \bar{\varphi}^i Q_{(-)i}, \phi_{(+)a} = \sigma^i Q_{(+)ai}, \phi_{(-)a} = \bar{\sigma}^i Q_{(-)ai}\,,
\end{aligned}
\tag{5.16}
$$

with $\Lambda = \Lambda' = \bar{\Lambda} = 1$. Eventually the potential (5.14) written in terms of the scalar fields (5.16) takes the form

$$
\begin{aligned}
V = & \left(\frac{8}{R_1^4} + \frac{4}{R_2^4}\right) - \frac{4}{R_1^2}(\sigma^i\bar{\sigma}_i) - \frac{8}{R_2^2}(\varphi^i\bar{\varphi}_i) + \frac{19}{3}(\varphi^i\bar{\varphi}_i)^2 + \\
& + \frac{16}{3}(\sigma^i\bar{\sigma}_i)^2 + \frac{1}{3}(\bar{\varphi}_i\bar{\varphi}_i)(\varphi^j\varphi^j) + \frac{1}{3}(\bar{\sigma}_i\bar{\sigma}_i)(\sigma^j\sigma^j)\,. \qquad (5.17)
\end{aligned}
$$

In order to examine further the present model we should employ again the flux mechanism and study the minimization of the potential (5.17). To keep unchanged the number of families background gauge configurations with non-trivial winding number are required as in the example discussed in section 4.

6. Conclusions

In the present work we have presented the details of the dimensional reduction of general ten-dimensional $\mathcal{N} = 1$ supersymmetric theories over coset spaces using the superfield formulation of the theory.

In this way we have extended and generalized our previous results concerning the supersymmetry breaking of ten-dimensional theories in the process of dimensional reduction found using the components formalism and a case by case study. Specifically we have shown that $\mathcal{N} = 1$, ten dimensional supersymmetric YM theories lead in four dimensions, after reduction over coset spaces, either to a softly broken $\mathcal{N} = 1$ supersymmetric theory if the cosets used are non-symmetric or to a non-supersymmetric theory if the cosets used are symmetric.

We have also presented two very interesting three family models resulting from a $\mathcal{N} = 1$ supersymmetric E_8 gauge theory in ten dimensions. The first one was reduced over the non-symmetric coset space $\mathrm{SU}(3)/\mathrm{U}(1) \times \mathrm{U}(1)$ and led in four dimensions to a softly broken $\mathcal{N} = 1$ gauge theory based on E_6-type gauge group. This model has been examined earlier using the components formalism and was reexamined here using superfields. The second model concerns the reduction of the same $\mathcal{N} = 1$, ten-dimensional E_8 gauge theory over the symmetric coset space $CP^2 \times S^2$. The resulting four-dimensional gauge theory is non-supersymmetric and is based on a SO(10)-type gauge group. The details of this dimensional reduction are presented for the first time.

Finally we have pointed out how the above very interesting three family models at the GUT scale can be broken further using the Wilson flux mechanism, while keeping the multiplicity of chiral fermions. We plan to discuss the details of the phenomenological analysis of both models in a future publication.

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